Lectured By Dr. H. K. Yadav.
Deptt. of Mathematics. Marwari College, DBD.
B.Sc. - Class - XII Relations and Functions, Date - 22-05-2020
Ex - 1.2.

Q.1: Show that the function $f: R_* \to R_*$ defined by $f(x) = \frac{1}{x}$ is one-one, where R_* is the set of all non-zero real numbers. Is the result true, if the domain R_* is replaced by N with codomain being same as R_*?

Soln: It is given that $f: R_* \to R_*$ is defined by $f(x) = \frac{1}{x}$.

For one-one, $f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

$\therefore$ f is one-one.

For onto. It is clear that for $y \in R_*$, there exists $x = \frac{1}{y} \in R_*$ (Exists as $y \neq 0$) such that

$$f(x) = \frac{1}{\left(\frac{1}{y}\right)} = \frac{1}{1} \times \frac{y}{1} = y.$$

$\therefore$ f is onto. Thus, the given function (f) is one-one and onto.

Now, Let us consider function $g: N \to R_*$ defined by $g(x) = \frac{1}{x}$

We have, $g(x_1) = g(x_2) \Rightarrow \frac{1}{x_1} = \frac{1}{x_2} \Rightarrow x_1 = x_2$.

$\therefore$ g is one-one.

Further, it is clear that g is not onto as for $1.2 \in R_*$, there does not exist any x in N such that $g(x) = \frac{1}{1.2}$

Hence, function g is one-one but not onto.

Q.2. Check the injectivity and surjectivity of the following functions.

(I) $f: N \to N$ given by $f(x) = x^2$

(II) $f: Z \to Z$ given by $f(x) = x^2$

(iii) $f: R \to R$ given by $f(x) = x^2$ ~~(iv) $f: N \to N$ given by $f(x) = x^3$~~
~~(v) $f: Z \to Z$ given by $f(x) = x^3$~~.

Soln: (I) $f: N \to N$ is given by $f(x) = x^2$
It is seen that for $x, y \in N$, $f(x) = f(y)$
$\Rightarrow x^2 = y^2 \Rightarrow x = y$ [$\because x$ and y are +ve nos.]
$\therefore$ f is injective.
Now, $2 \in N$, but there does not exist any x in N such that $f(x) = x^2 = 2$
It means there is some element in co-domain in which do not have any images. Therefore, f is not surjective.
Hence, function f is injective but not surjective.

(II) $\because f: Z \to Z$ is given by $f(x) = x^2$
It is seen that $f(-1) = f(1) = 1$ but $-1 \neq 1$.
$\therefore$ f is not injective.
Now, $-2 \in Z$. But there does not exist any element $x \in Z$ such that $f(x) = x^2 = -2$.
$\therefore$ f is not surjective.
Hence, function f is neither injective nor surjective.

(III) $\because f: R \to R$ is given by $f(x) = x^2$. It is seen that $f(-1) = f(1) = 1$ but $-1 \neq 1$.
$\therefore$ f is not injective.
Now, $-2 \in R$. But there does not exist any element $x \in R$ such that
$f(x) = x^2 = -2$. Therefore, f is not surjective.
Hence, function f is neither injective nor surjective.